Name: Date: Period: 1

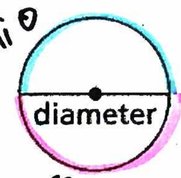
Chords of Circles

Lesson Objective IWBAT use chords of ⊙s to find ~~lem~~ lengths of segments and arc measures

Remember that a chord is a segment with endpoints on the circle.

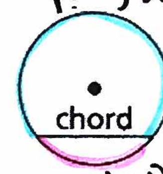
Because the endpoints lie on the circle, any chord will divide the circle into two arcs:

semi ⊙



semi ⊙

major arc

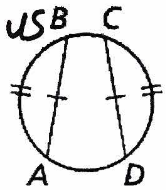


minor arc

Congruent Corresponding Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

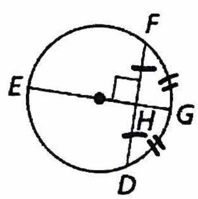
→ same radius



minor arcs $\cong$ $\leftrightarrow$ chords $\cong$

Perpendicular Chord Bisector Theorem

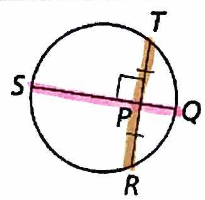
If a diameter (or radius) of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.



diameter $\perp$ to chord → ① bisects chord → ② bisects arc

Perpendicular Chord Bisector Converse

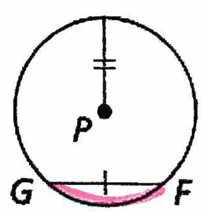
If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.

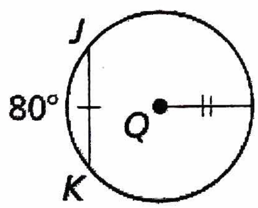


$chord_1$ $\perp$ bisects $chord_2$ → $chord_1$ is a diameter

Example:

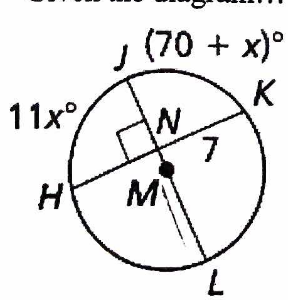
1. In the diagram, $\odot P \cong \odot Q, \overline{FG} \cong \overline{JK}$ and $m\widehat{JK} = 80°$. Find $m\widehat{FG}$.





80° chords $\cong$ → minor arcs $\cong$

2. Given the diagram…



a. Find HK

HN = 7

$\boxed{HK = 14}$

b. Find $m\widehat{HK}$

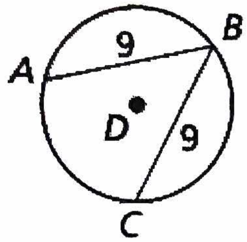
$11x = 70 + x$

$10x = 70$

$x = 7$

$m\widehat{HK} = 2(11(7)) = \boxed{154°}$

3. Given the diagram…



a. If $m\widehat{AB} = 110°$, what is $m\widehat{BC}$?

$110°$ chords $\cong \rightarrow$ minor arcs $\cong$

b. If $m\widehat{AC} = 150°$, what is $m\widehat{AB}$?

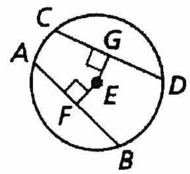
$150 + x + x = ~~180~~\ 360$

$2x = 210 \rightarrow x = 105$ $\boxed{105°}$

Equidistant Chords Theorem

(same radius)

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

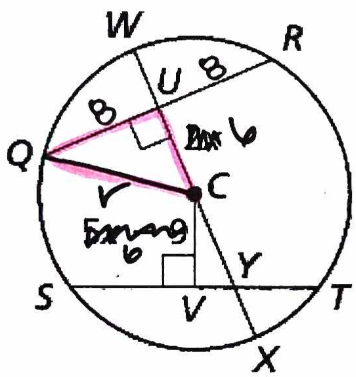


Chords $\cong$ $\leftrightarrow$ dist. to center $\cong$ ($\perp$)

Example

5. In the diagram, QR = ST = 16, CU = 2x, and CV = 5x – 9. Find the radius of $\odot C$.

$\rightarrow$ Chord $\cong$ $\rightarrow$ dist. to center $\cong$ (CV = CU)



* $2x = 5x - 9$

$3x = 9$

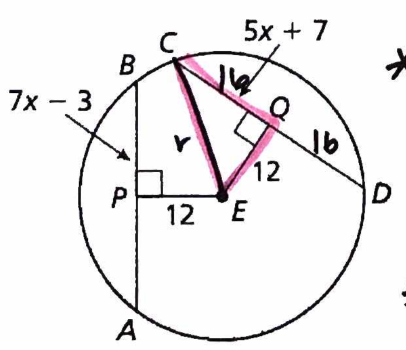
$x = 3$

* $QU = \frac{1}{2}(16) = 8$ (diam. $\perp$ to chord $\rightarrow$ bisects chord)

* $8^2 + 6^2 = r^2$

$\boxed{r = 10}$

6. In the diagram, EP = EQ = 12, CD = 5x + 7 and AB = 7x – 3. Find the radius of $\odot E$.



* $5x + 7 = 7x - 3$ (dist. to center $\cong$ $\rightarrow$ chords $\cong$)

$10 = 2x$

$5 = x$

* $CD = 32$

* $CQ = 16$ (diam. $\perp$ to chord $\rightarrow$ bisects chord)

* $12^2 + 16^2 = r^2 \rightarrow$ $\boxed{r = 20}$